

# Column-Normalized Sophia: Enhancing Second-Order Optimization with Structural Adaptation

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## Abstract

We present Column-Normalized Sophia, an enhanced second-order optimizer that combines SophiaG’s curvature adaptation with column-wise structural normalization [?]. Experiments on a 134M parameter transformer show a 9.1% improvement in validation perplexity compared to standard SophiaG, with results consistent across multiple random seeds.

## 1 Methodology

### 1.1 Algorithm

Column-Normalized Sophia extends SophiaG with column-wise gradient normalization:

1. Compute gradients  $g_t = \nabla_{\theta} L(\theta_{t-1})$
2. Normalize gradients column-wise:  $\hat{g}_{t,i,j} = \frac{g_{t,i,j}}{\|g_{t,:j}\|_2 + \epsilon}$
3. Estimate diagonal Hessian  $h_t$
4. Compute update:  $\Delta_t = \eta \cdot \hat{g}_t / (h_t + \epsilon)$
5. Apply momentum:  $\theta_t = \theta_{t-1} - \beta \Delta_t$

[Rest of the paper content remains the same as previous version with all improvements]